An Arts and Mathematics interaction based on paintings involving Ulam's spiral

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NITheCS
National Institute for
Theoretical and Computational Sciences

Paintings by Reg Alcorn

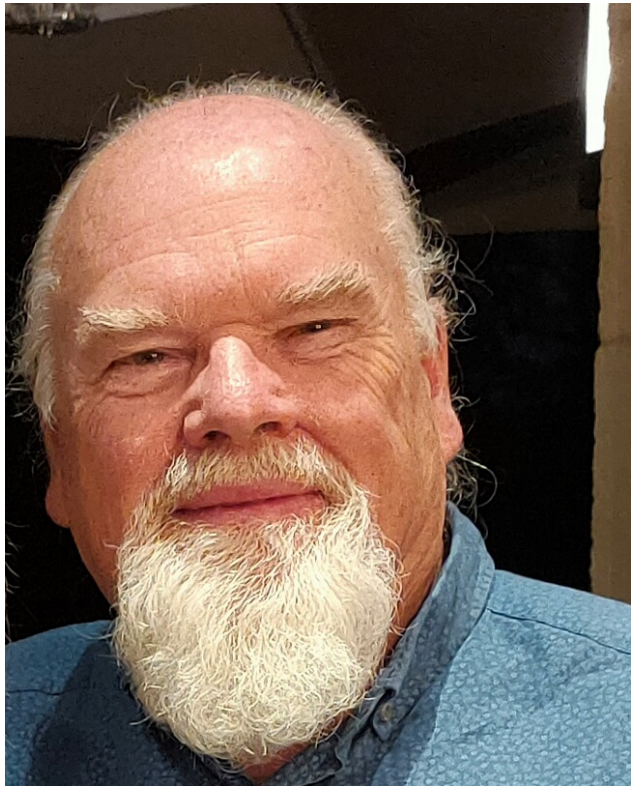
NITheCS Colloquium
Monday, 27 January 2025

Let All Blues Rejoice – Reg Alcorn (2019)

Reg Alcorn

Stéphane Vinatier & Reg Alcorn. "Les mathématiques vues par un artiste : des objets mathématiques qui inspirent". In *La Gazette de la Société Mathématique de France*, **181**, July 2024.

Reg Alcorn



William Reginald Alcorn was born in Zambia in 1950 to Irish and Scottish parents. He developed a passion for painting at a very early age. After studying humanities in England, he travelled around Europe, eventually settling in France. With projects in music, languages, science and, more recently, mathematics, he divides his time between his painting studio and popularising the arts, notably as part of the activities of the IREM of Limoges.

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The beginning of the story

Collaborations between scientists and the artist Reg Alcorn around actions of diffusion of sciences:

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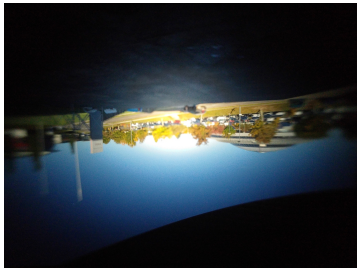

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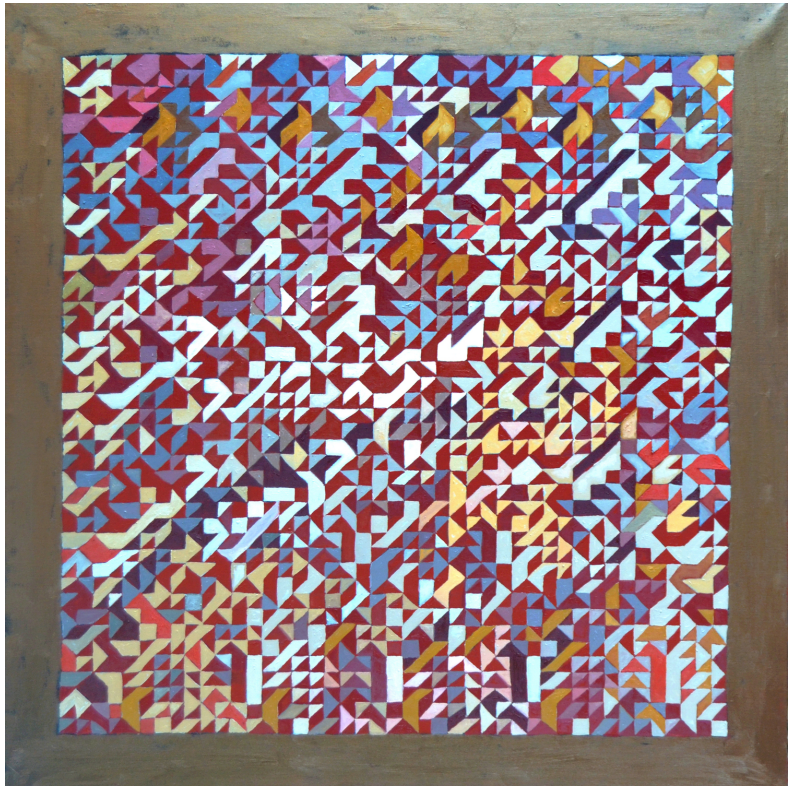
- The *Camera Obscura*
- exhibition *Research in Mathematics and Computer Science in Limoges as seen by an artist*




Poincaré's Breakfast (2012)

A desire for geometry

Like several other artists before him (Picasso, Mondrian, Kandinsky, Vasarely, Morellet...), Reg Alcorn was looking for a geometric process to structure the canvases of a series of paintings, that he would call *Transitions*:



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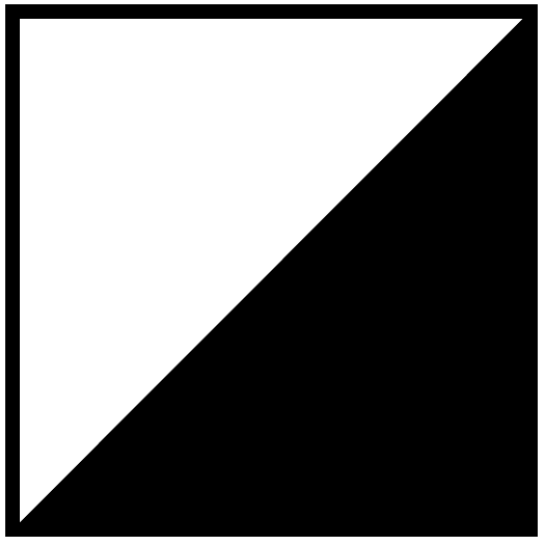

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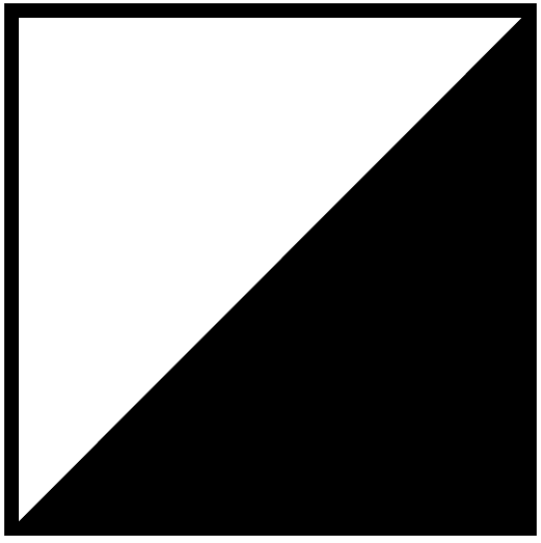
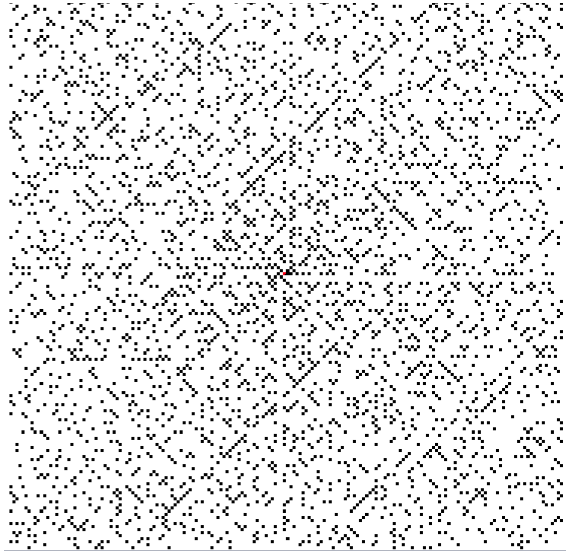


Truchet tiles



Ulam's spiral

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Truchet

Révérend Father Sébastien Truchet (1657–1729) was a mathematician, typographer, "clockmaker, great canal specialist, inventor of countless machines (cannons, tree-transplanting machines, sundials, etc.) including the famous mechanical tables of Marly"

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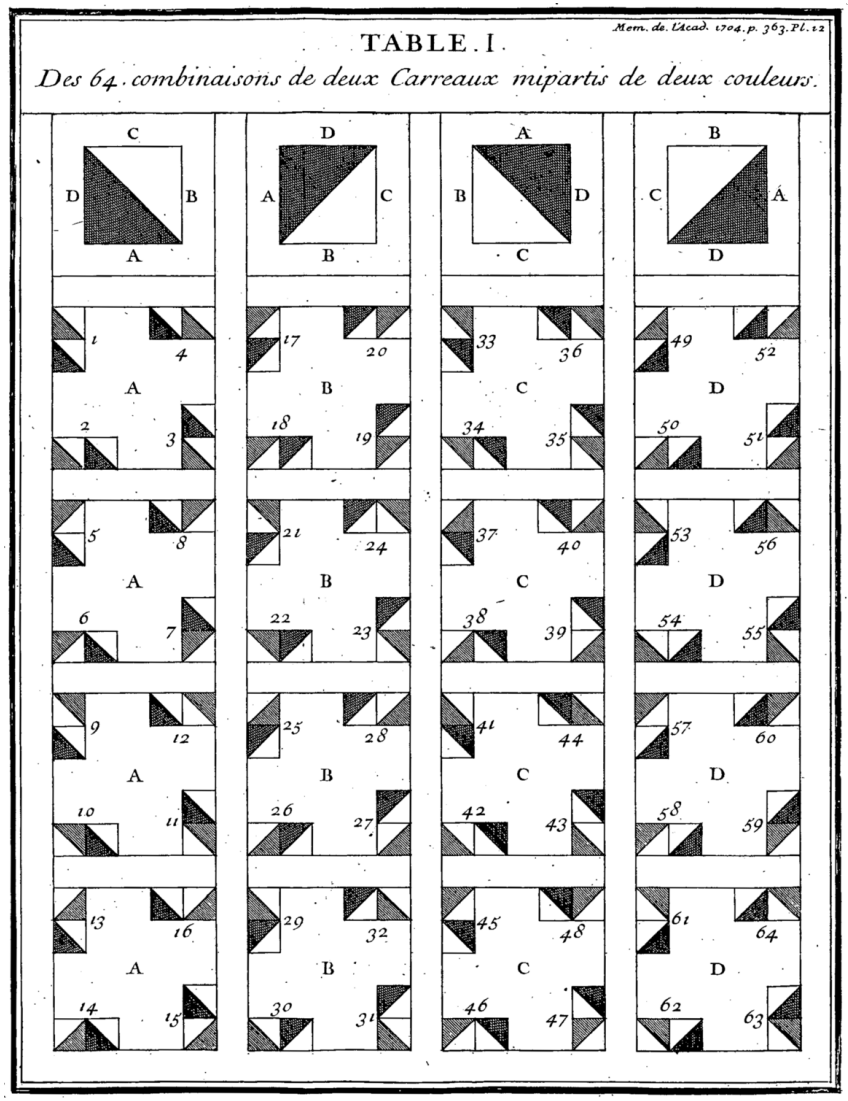
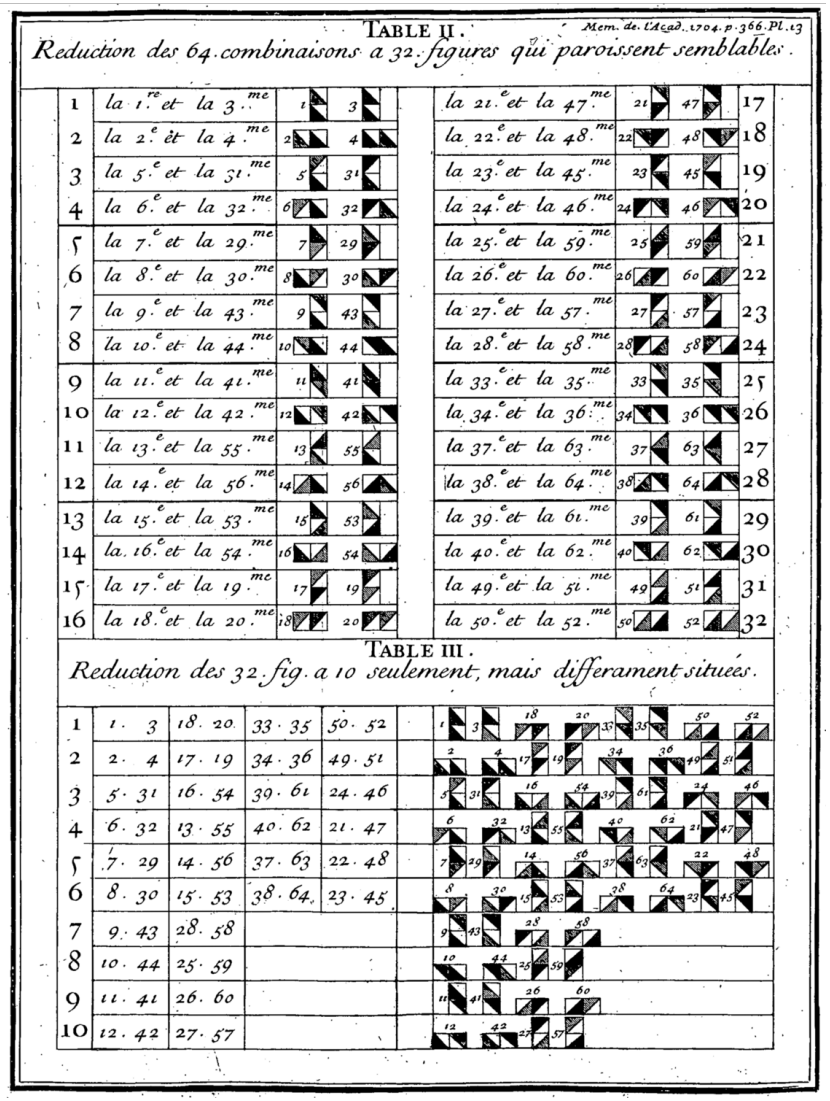
Truchet's treatise is of considerable importance for it is in essence a graphical treatment of combinatorics, a subject that, under the influence of Pascal, Fermat and Leibniz, was at the forefront of mathematics at the time.

Bicolor "carreaux mi-partis"





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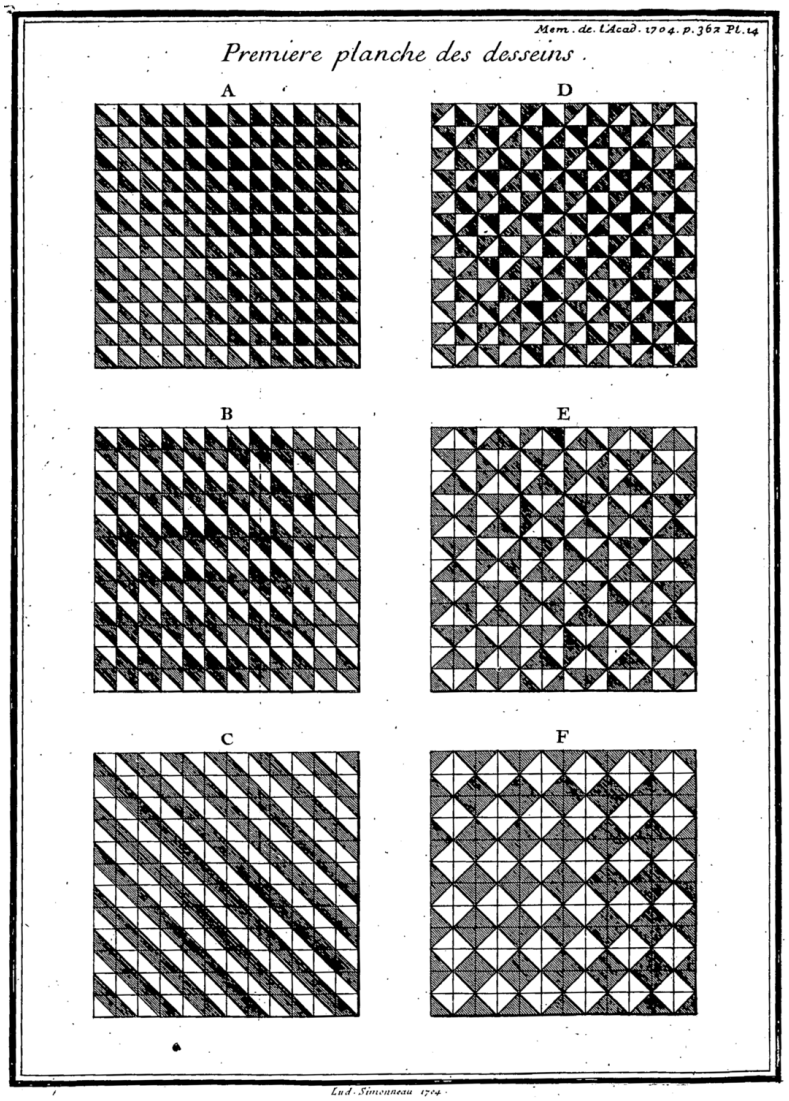

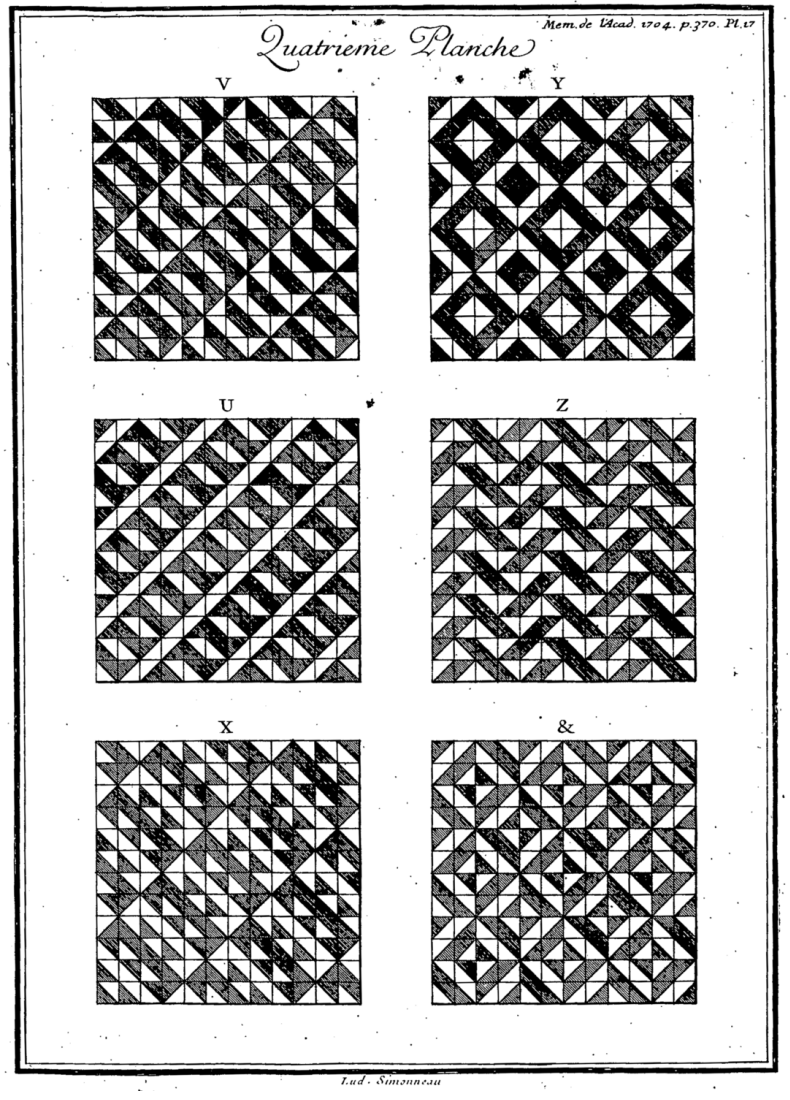

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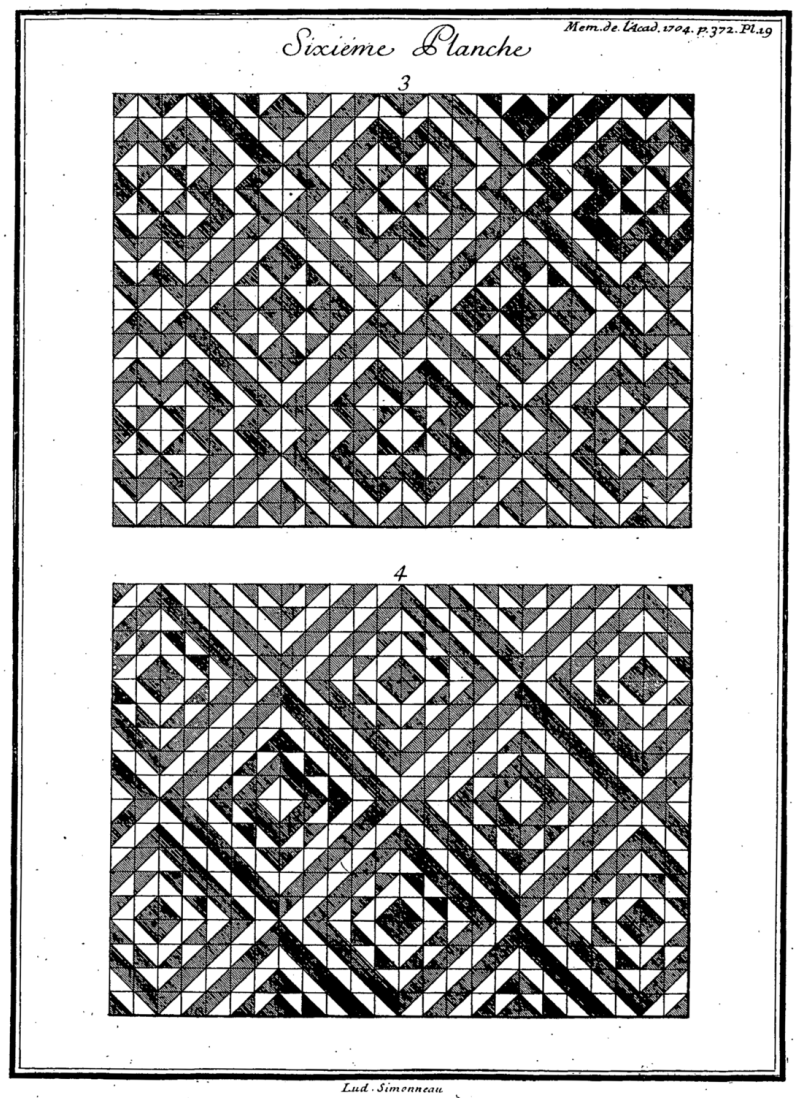

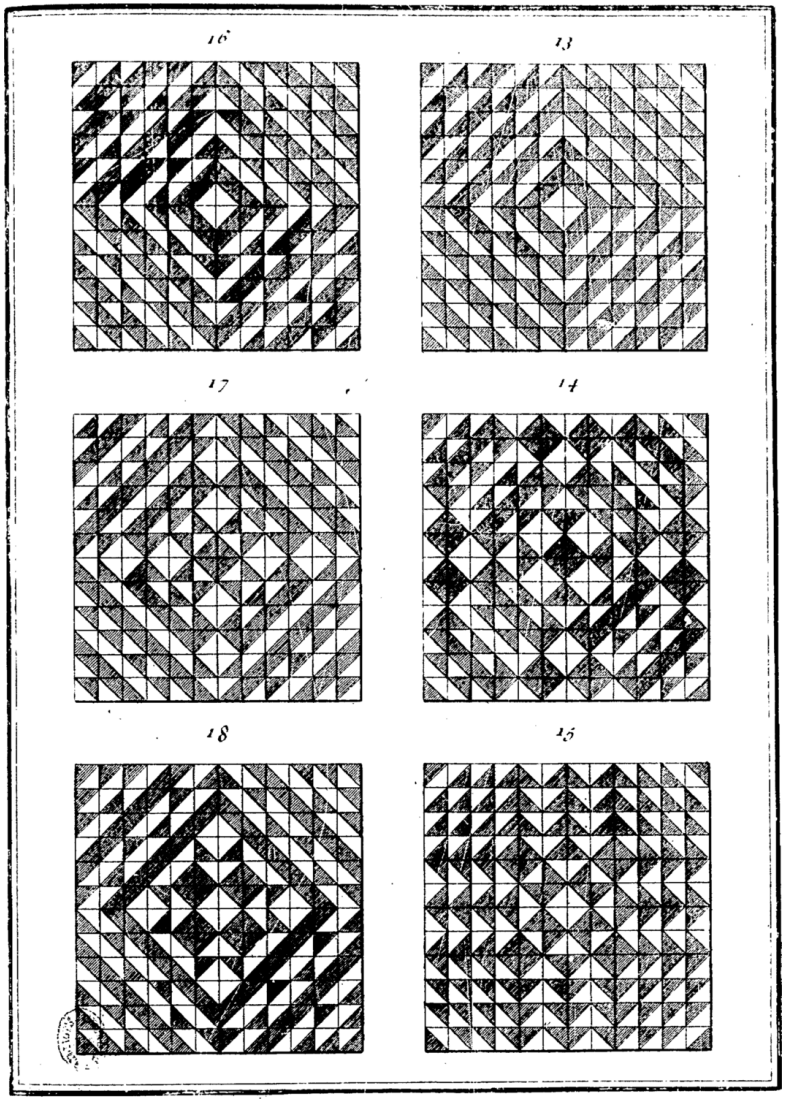

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Ulam

Stanislaw Ulam (1909-1984), was a polish mathematician, who migrated to the USA in the 30's. He joined the *Manhattan project* in 1943 and took part to the conception of the atomic bomb, at the Los Alamos laboratory; made a decisive contribution to the development of H bomb; made important contributions in several domains of fundamental mathematics.

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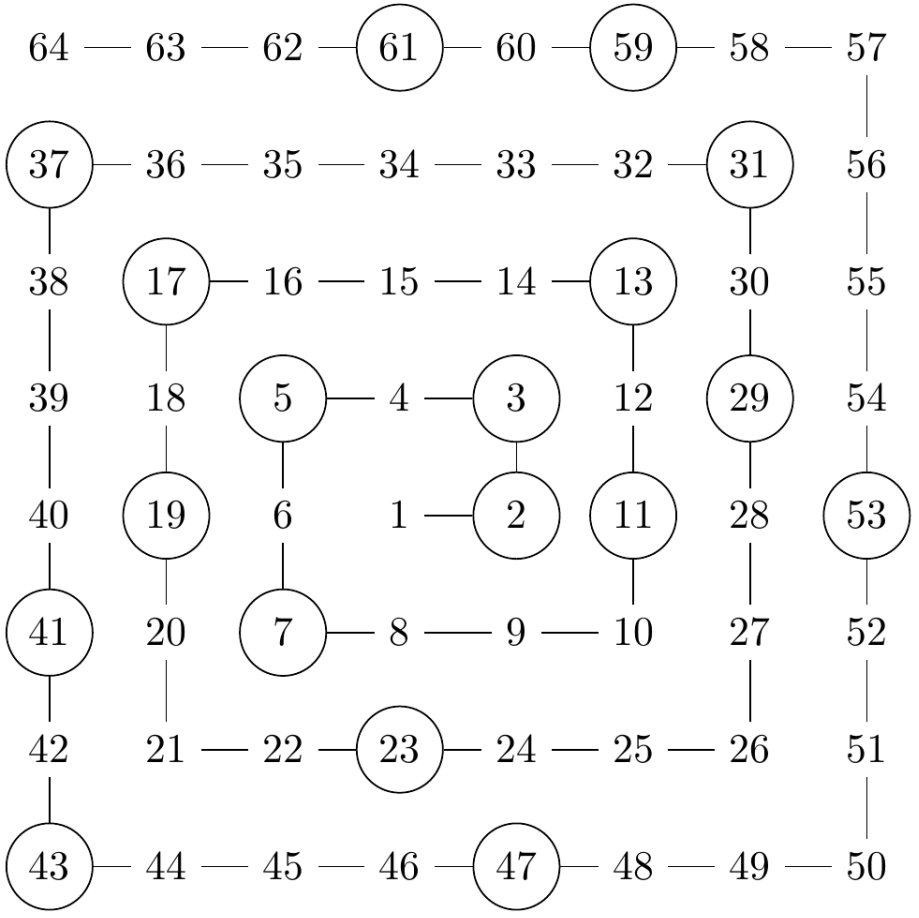
While getting bored during a conference in 1963, he began to draw a grid on a sheet of paper to represent a chessboard; changing his mind, he began to number the intersections starting from the center of the grid and spiraling around it in an anti-clockwise direction; then he began to circle the prime numbers and, to his great surprise, saw lines forming along the diagonals.

Ulam's spiral



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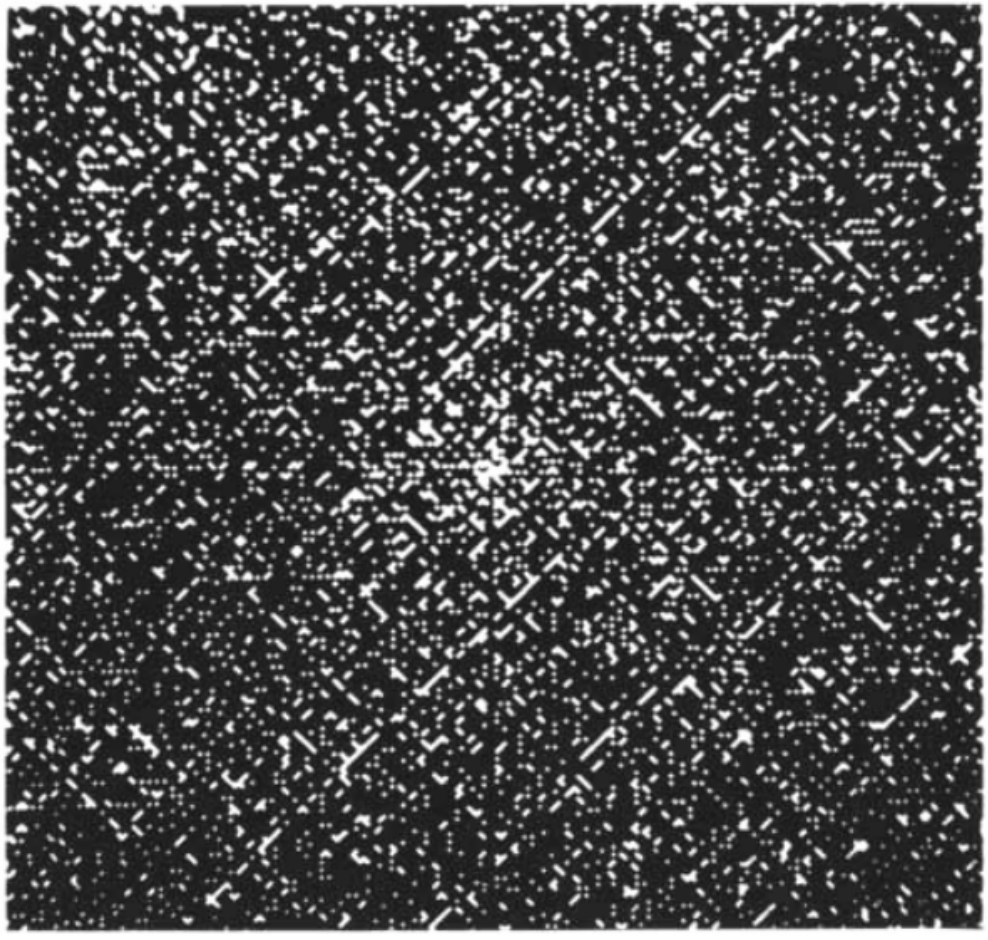

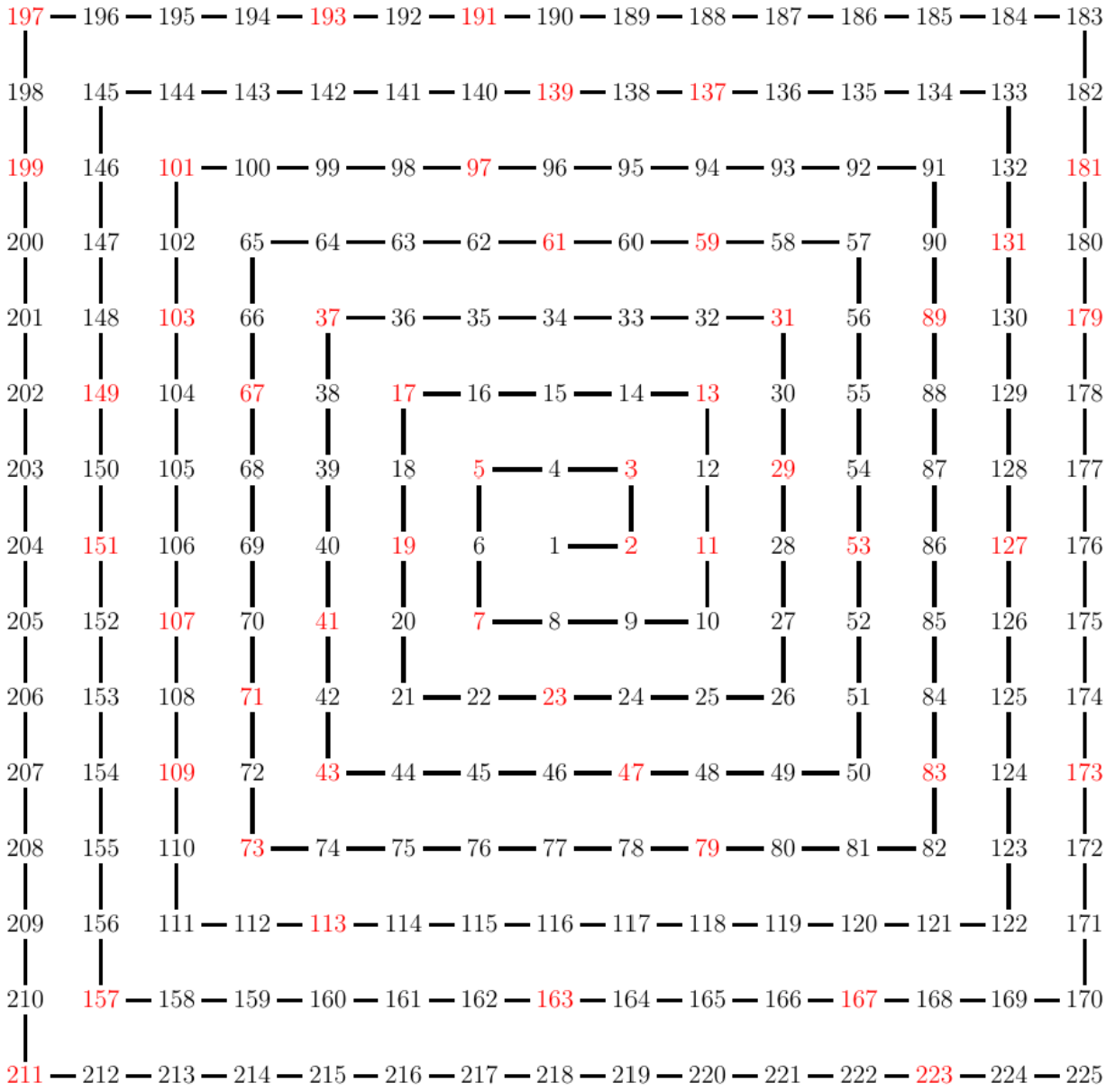
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200 147 102 65 64 63 62 61 60 59 58 57 90 131 180
201 148 103 66 37 36 35 34 33 32 31 56 89 130 179
202 149 104 67 38 17 16 15 14 13 30 55 88 129 178
203 150 105 68 39 18 5 4 3 12 29 54 87 128 177
204 151 106 69 40 19 6 1 2 11 28 53 86 127 176
205 152 107 70 41 20 7 8 9 10 27 52 85 126 175
206 153 108 71 42 21 22 23 24 25 26 51 84 125 174
207 154 109 72 43 44 45 46 47 48 49 50 83 124 173
208 155 110 73 74 75 76 77 78 79 80 81 82 123 172
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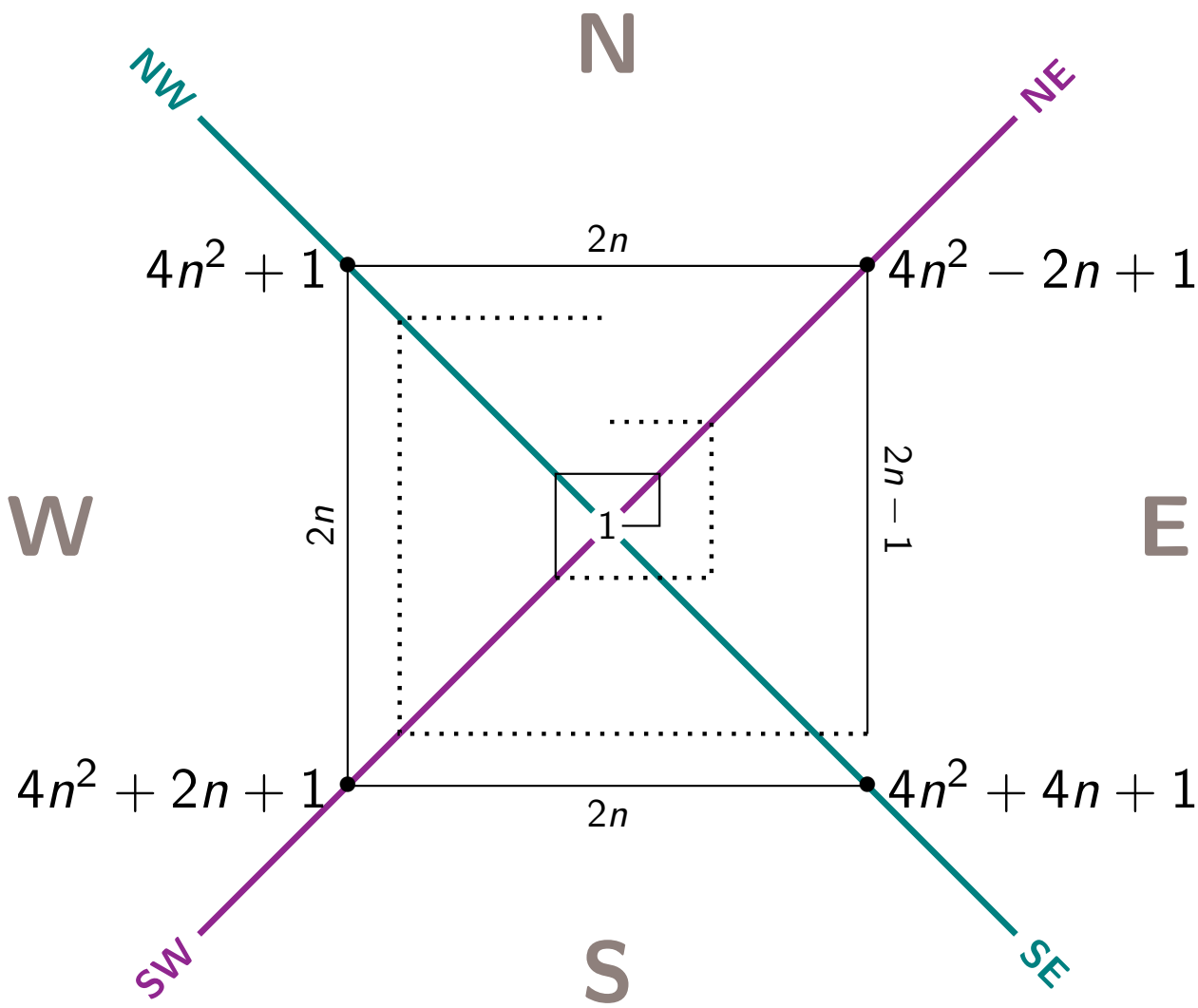
N
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E
SW
SE
S
$4n^2+1$
$2n$
$4n^2-2n+1$
$2n$
$2n-1$
1
$4n^2+2n+1$
$2n$
$4n^2+4n+1$

Euler's formula

The function

$$E(n) = n^2 + n + 41$$

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takes

- 40 distinct prime values at consecutive integers ($0 \leq n \leq 39$)
- 47.5% of prime values among values up to 10 000 000

Euler's formula

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$$E(n) = n^2 + n + 41$$

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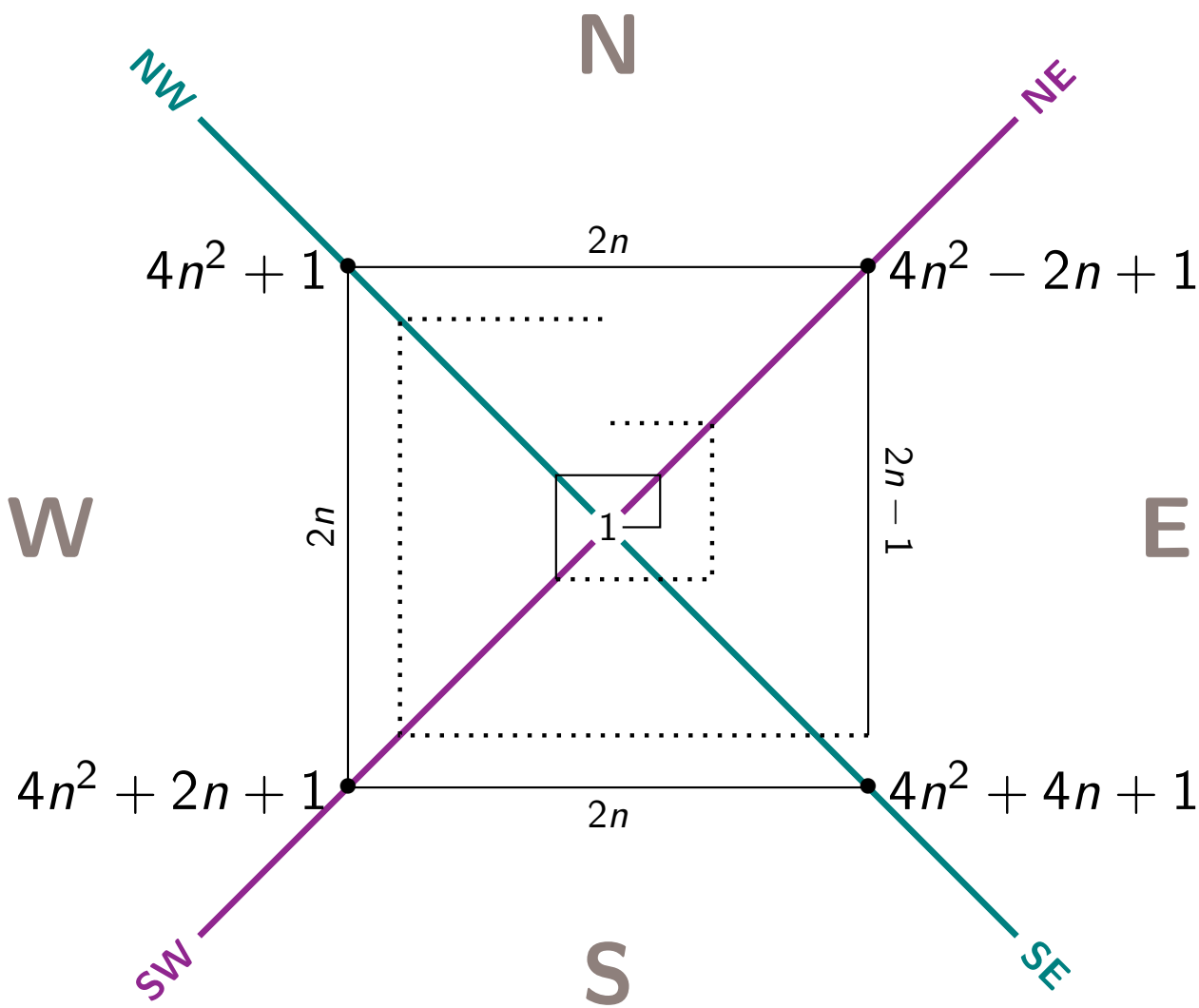
- 47.5% of prime values among values up to 10 000 000

One computes:

$$E(2n-1) = (4n^2 - 2n + 1) + 40$$
$$E(2n) = (4n^2 + 2n + 1) + 40$$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$4n^2-2n+1$
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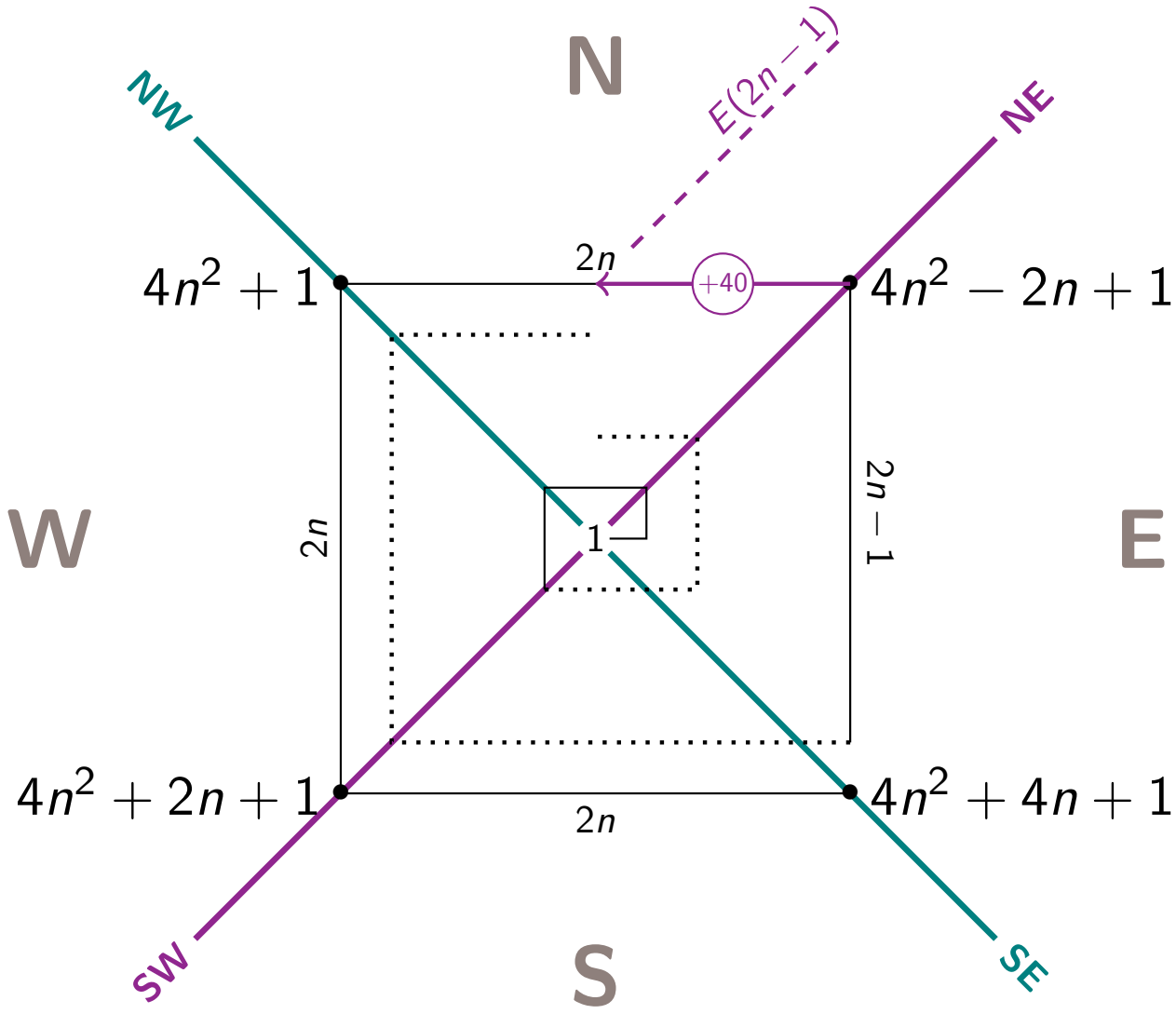
1
$4n^2+2n+1$
$2n$
$4n^2+4n+1$


N
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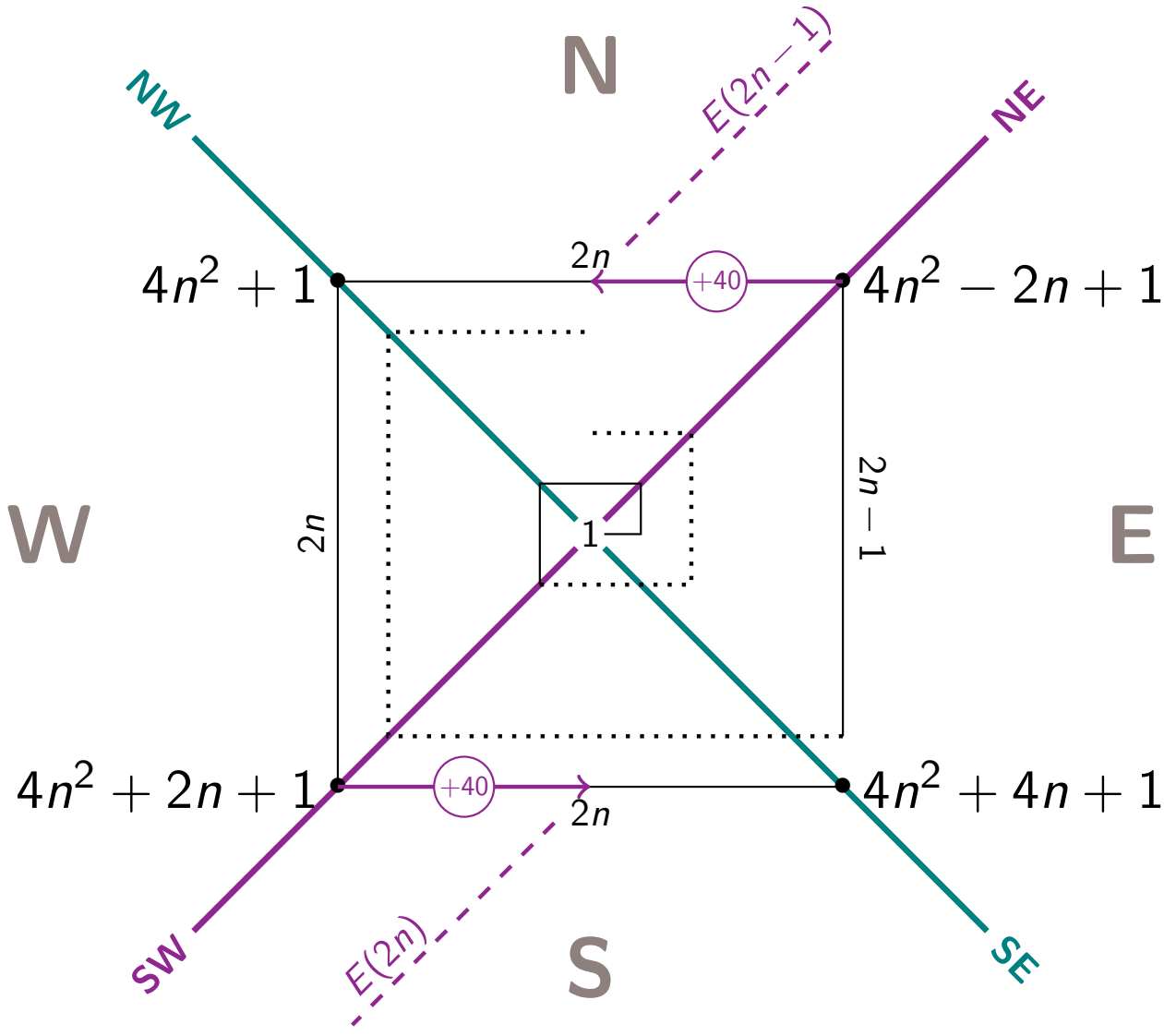
$E(2n-1)$
$4n^2+1$
$4n^2-2n+1$
$4n^2+2n+1$
$4n^2+4n+1$
$2n$
+40
$2n$
$2n-1$
1
$2n$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$E(2n-1)$
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$4n^2+1$
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$4n^2+2n+1$
$4n^2+4n+1$
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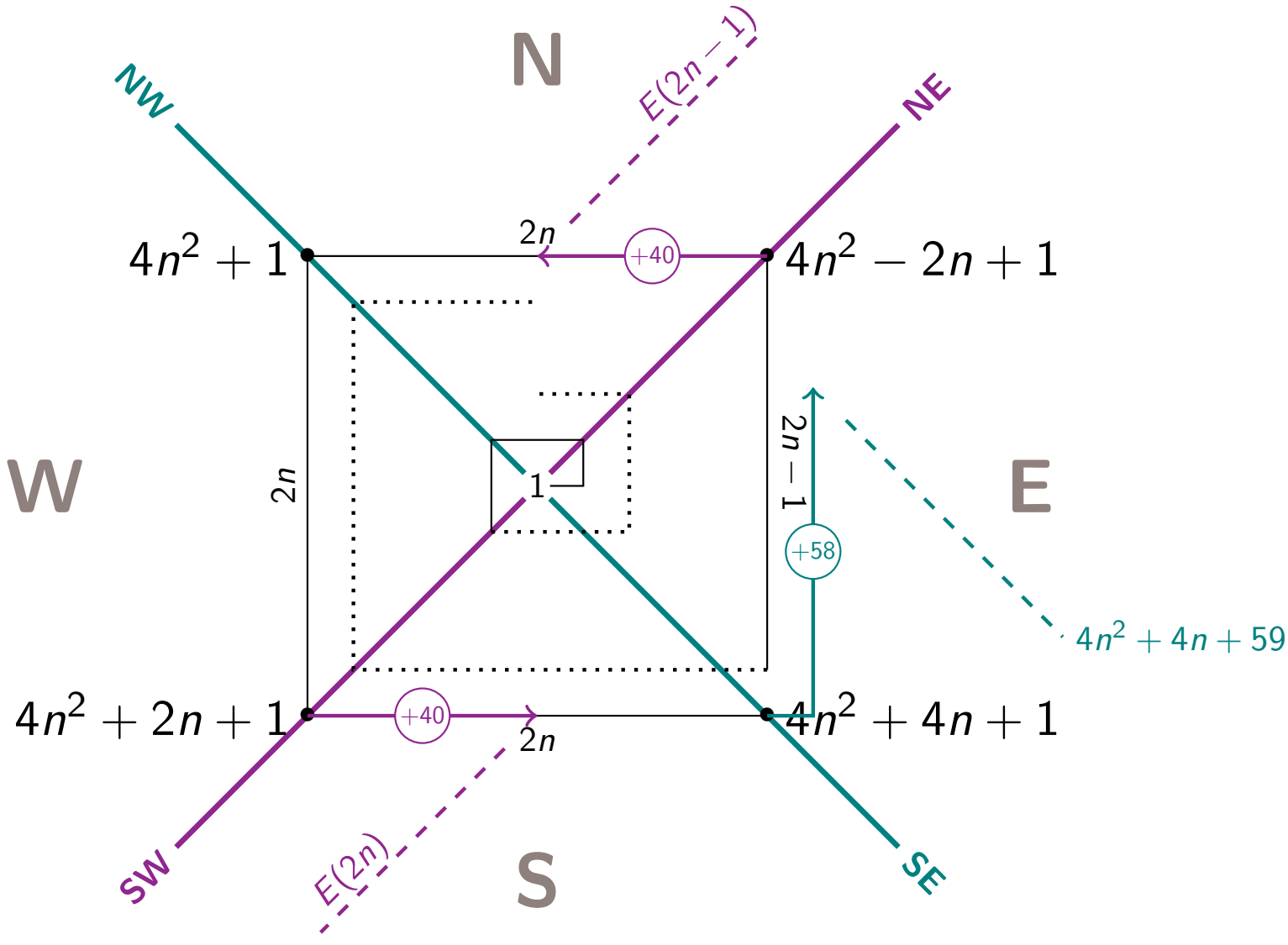
$2n$
$2n$
$2n-1$
1
+40
+40


N
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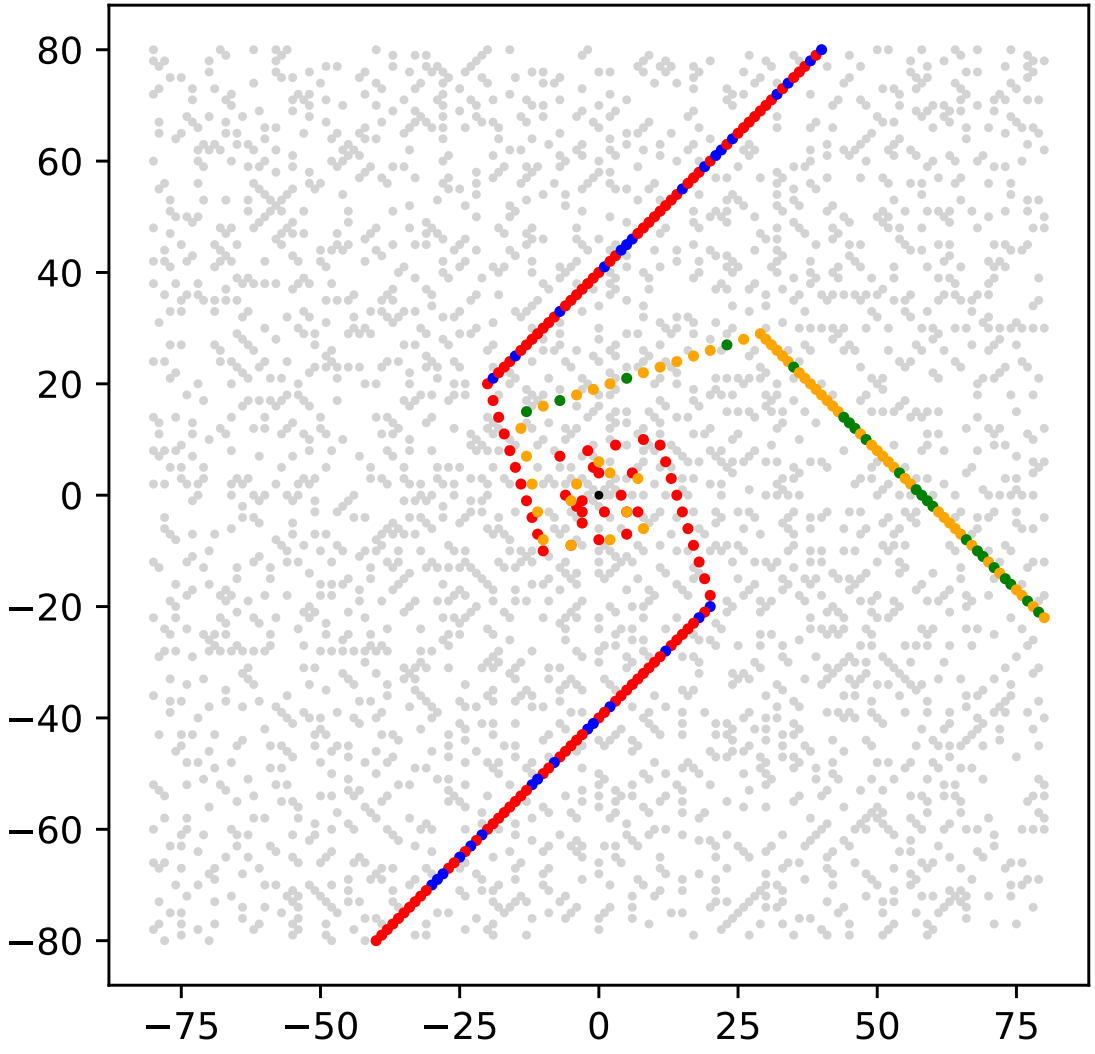
$4n^2+1$
$4n^2-2n+1$
$4n^2+2n+1$
$4n^2+4n+1$
$2n$
$2n$
$2n-1$
1
+40
+40
+58
$E(2n-1)$
$E(2n)$
$4n^2+4n+59$

Ulam and Euler and ...



$E(n)$ for $n \leq 160$:
prime values
other valeurs

$4n^2 + 4n + 59$
for $n \leq 80$:
prime values
other values

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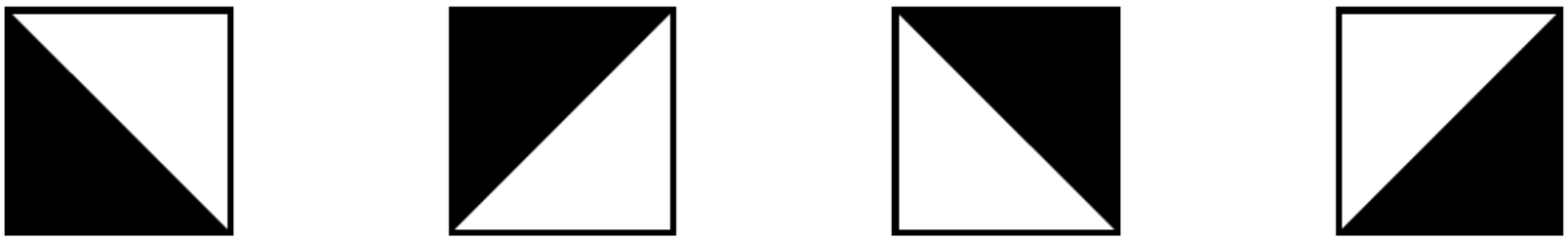
Truchet tiles

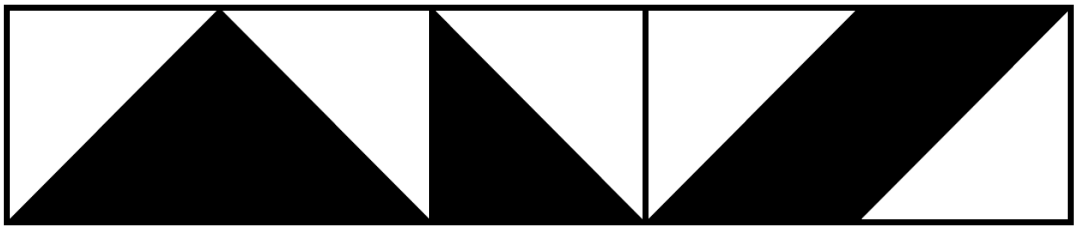


Truchet tiles

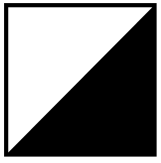
a b c d k w

Truchet tiles

Reg Alcorn chooses a sequence of tiles, for instance *daadb* :



daadb



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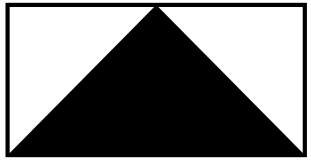

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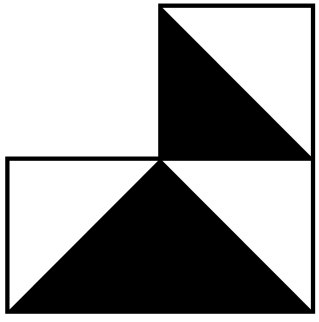

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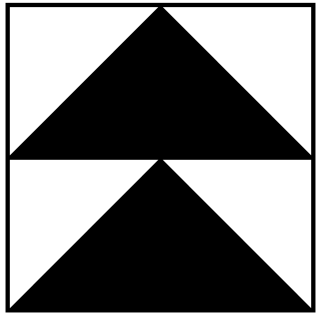

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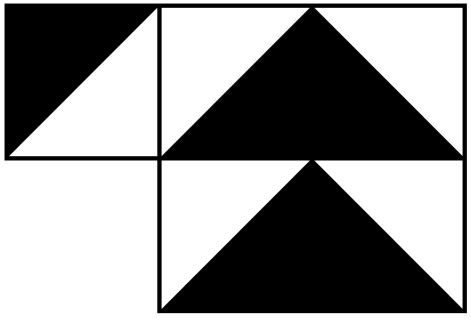

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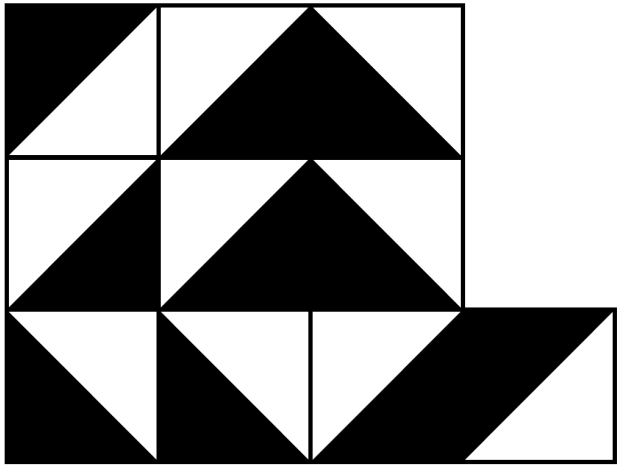

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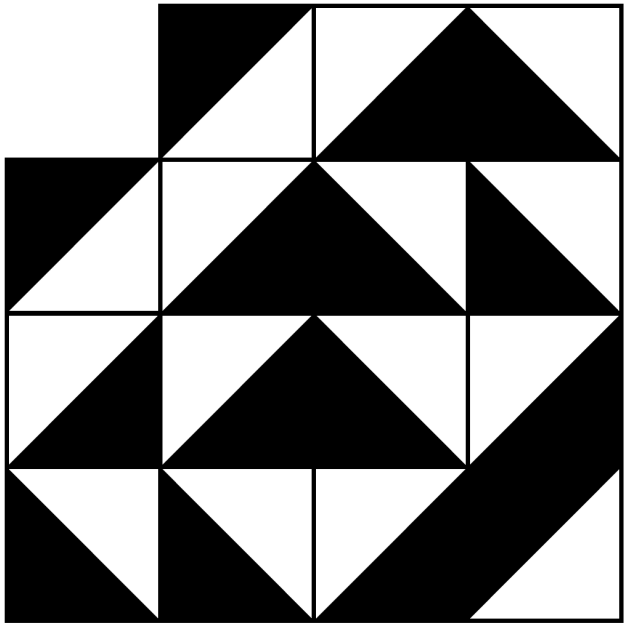

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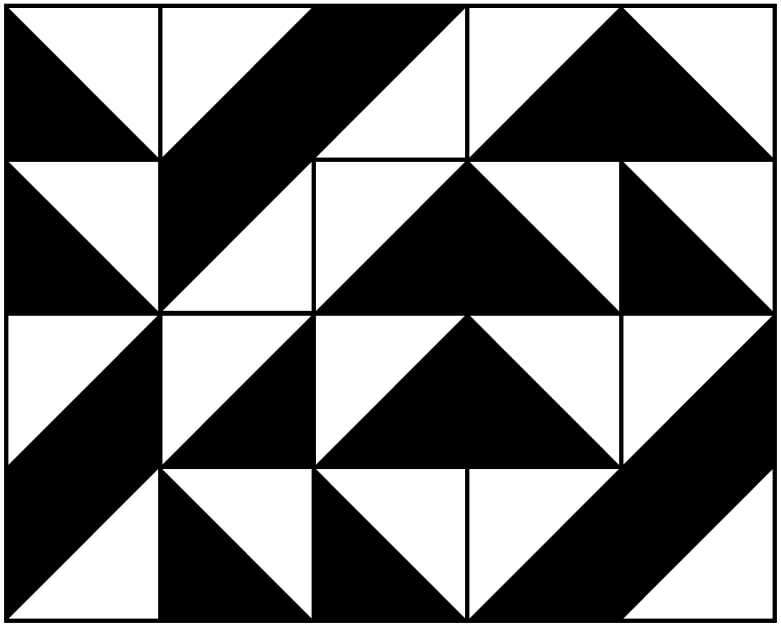

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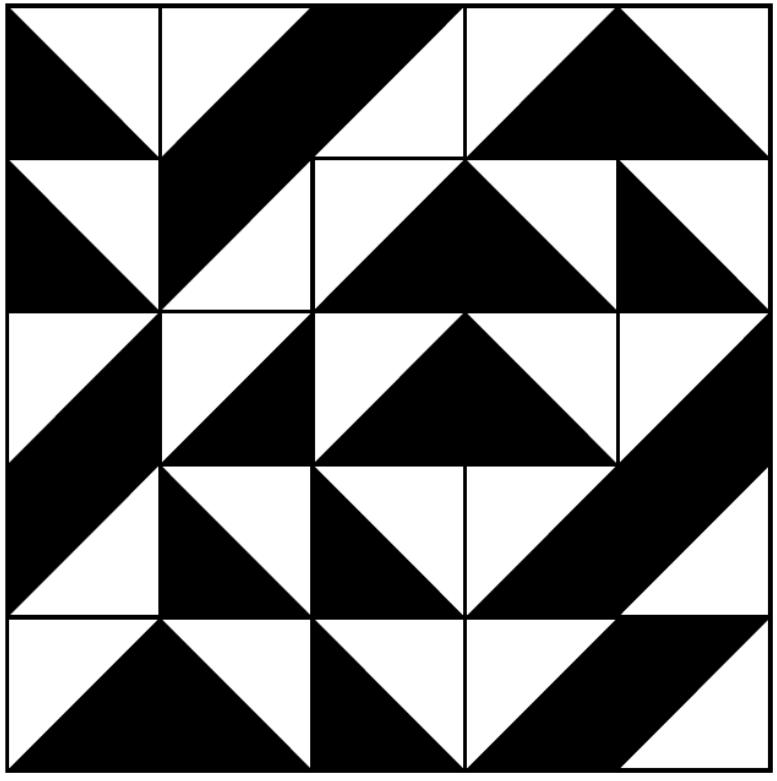

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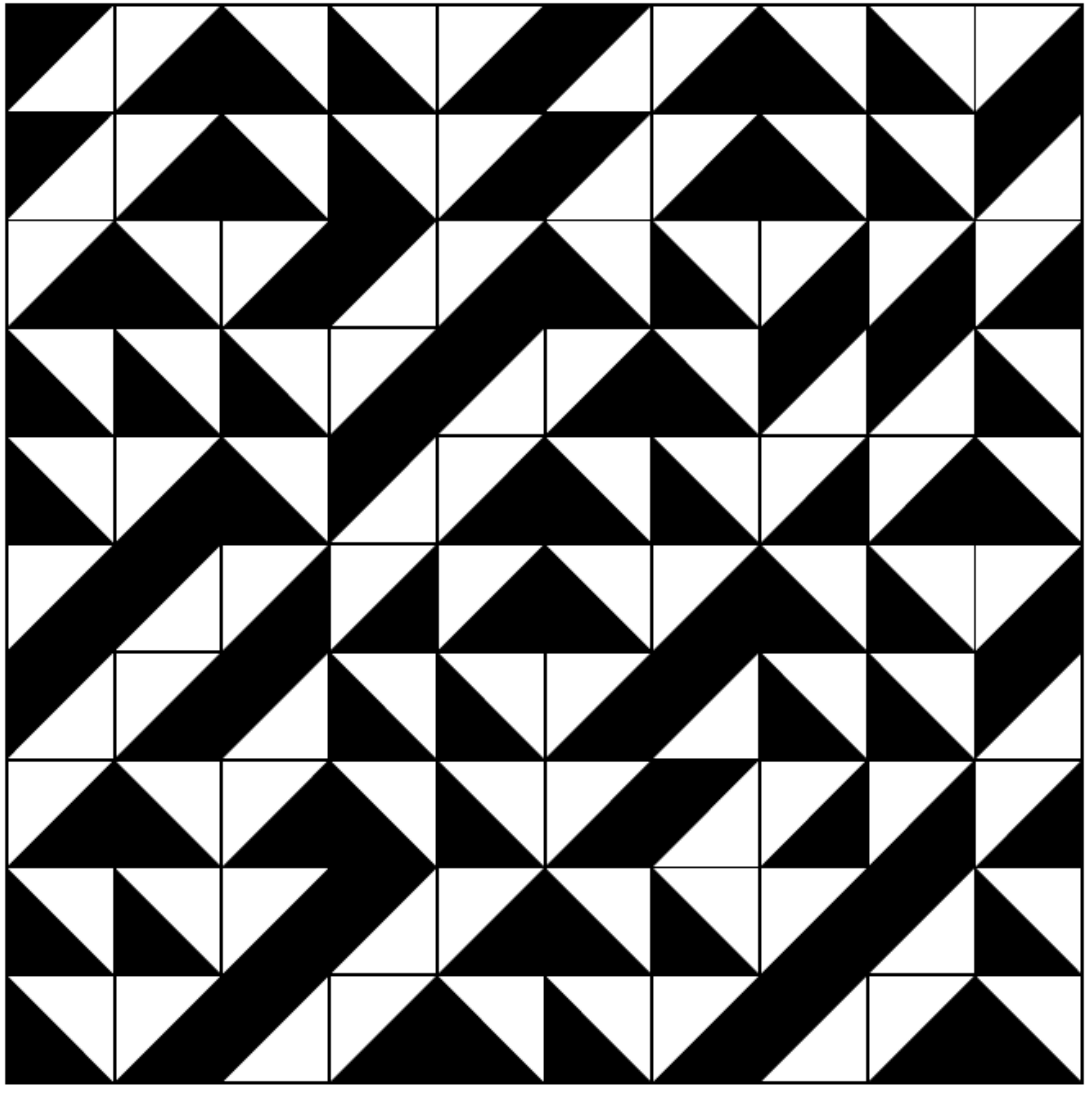
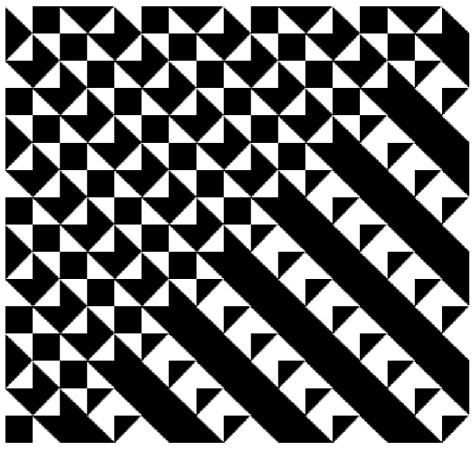
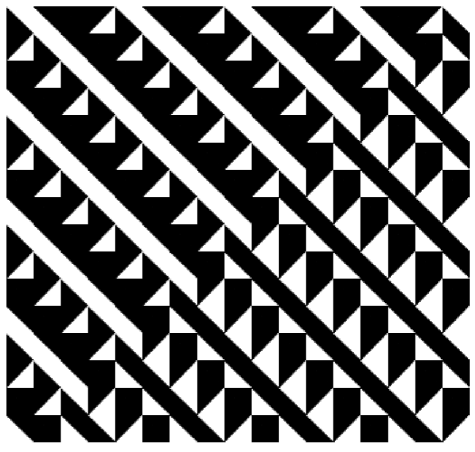
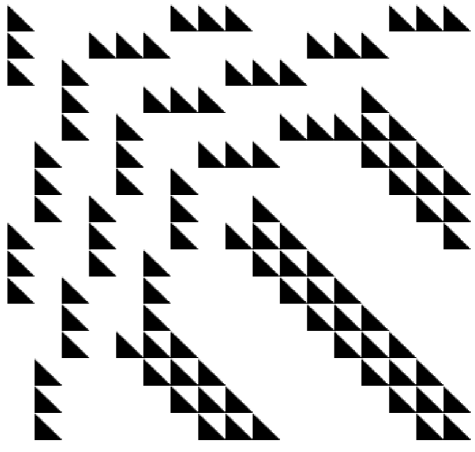


abck, *abkc* and *aaawwwww*







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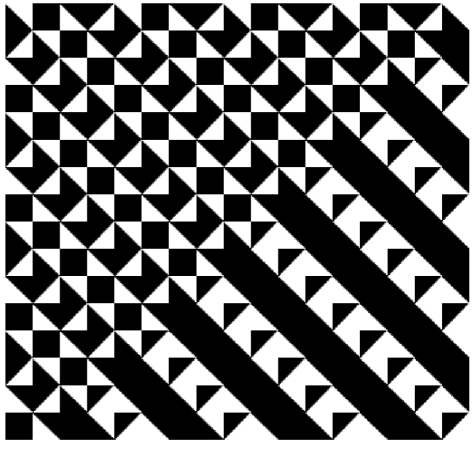
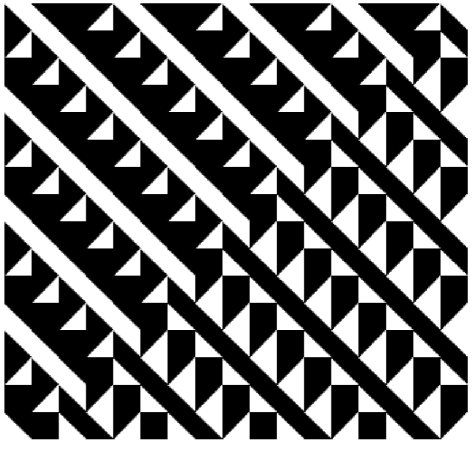
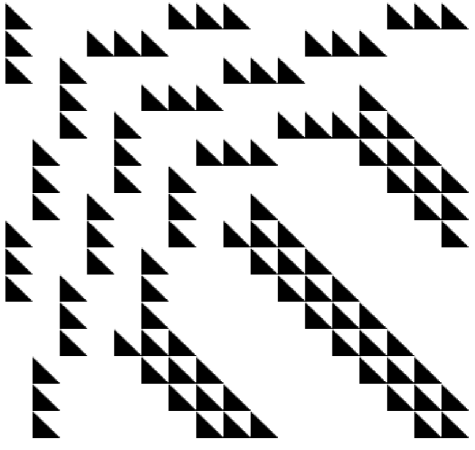
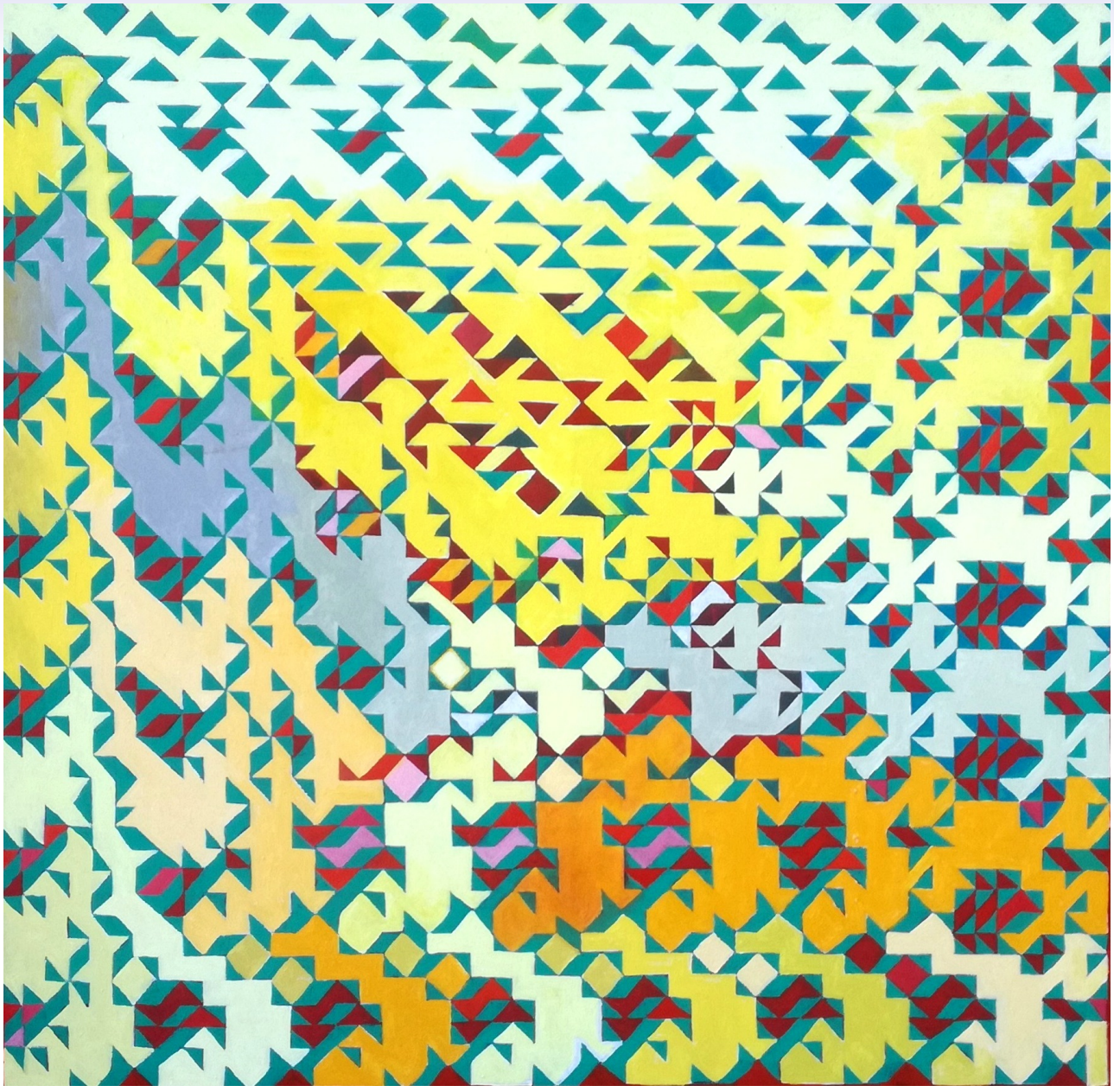
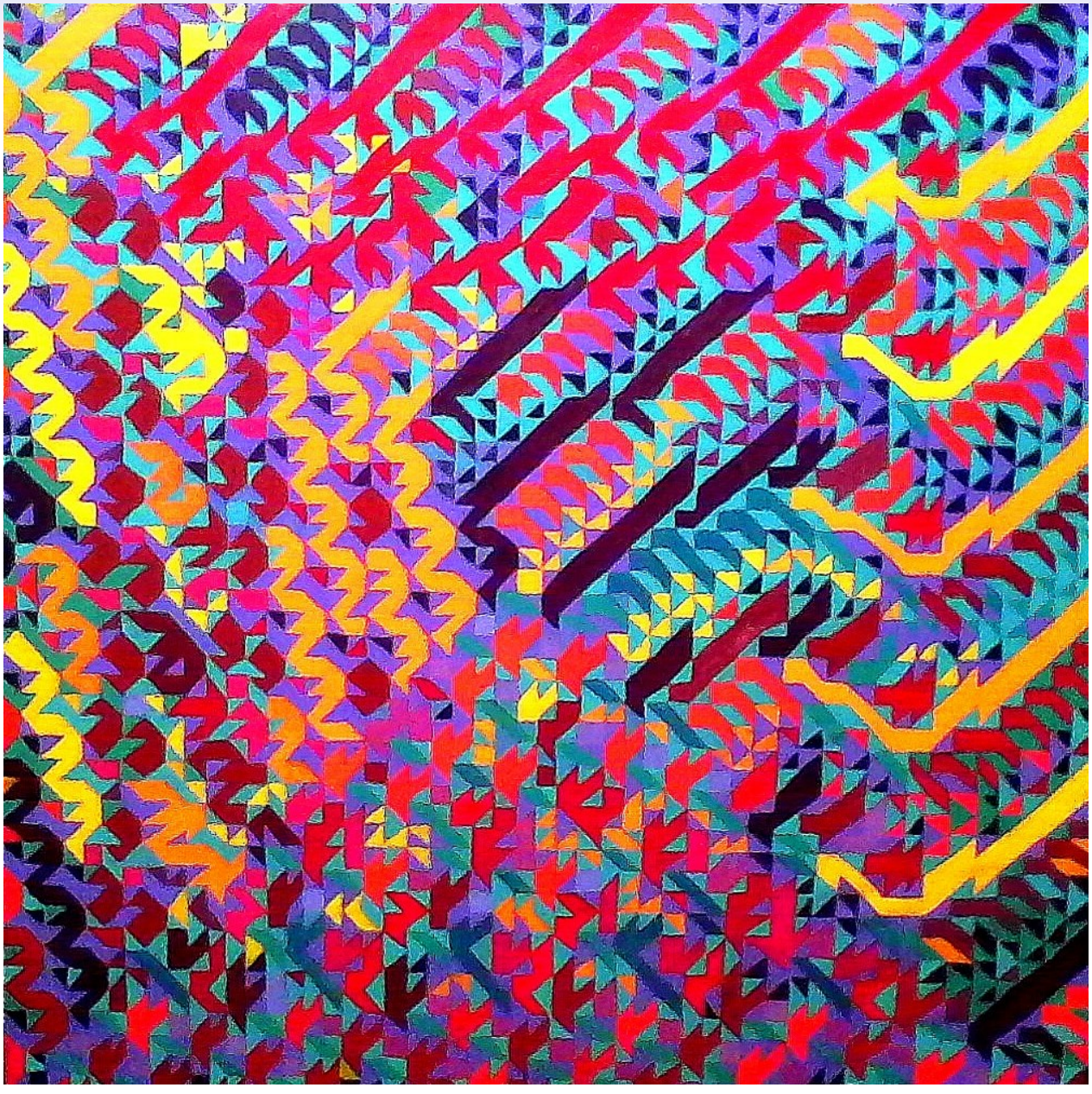




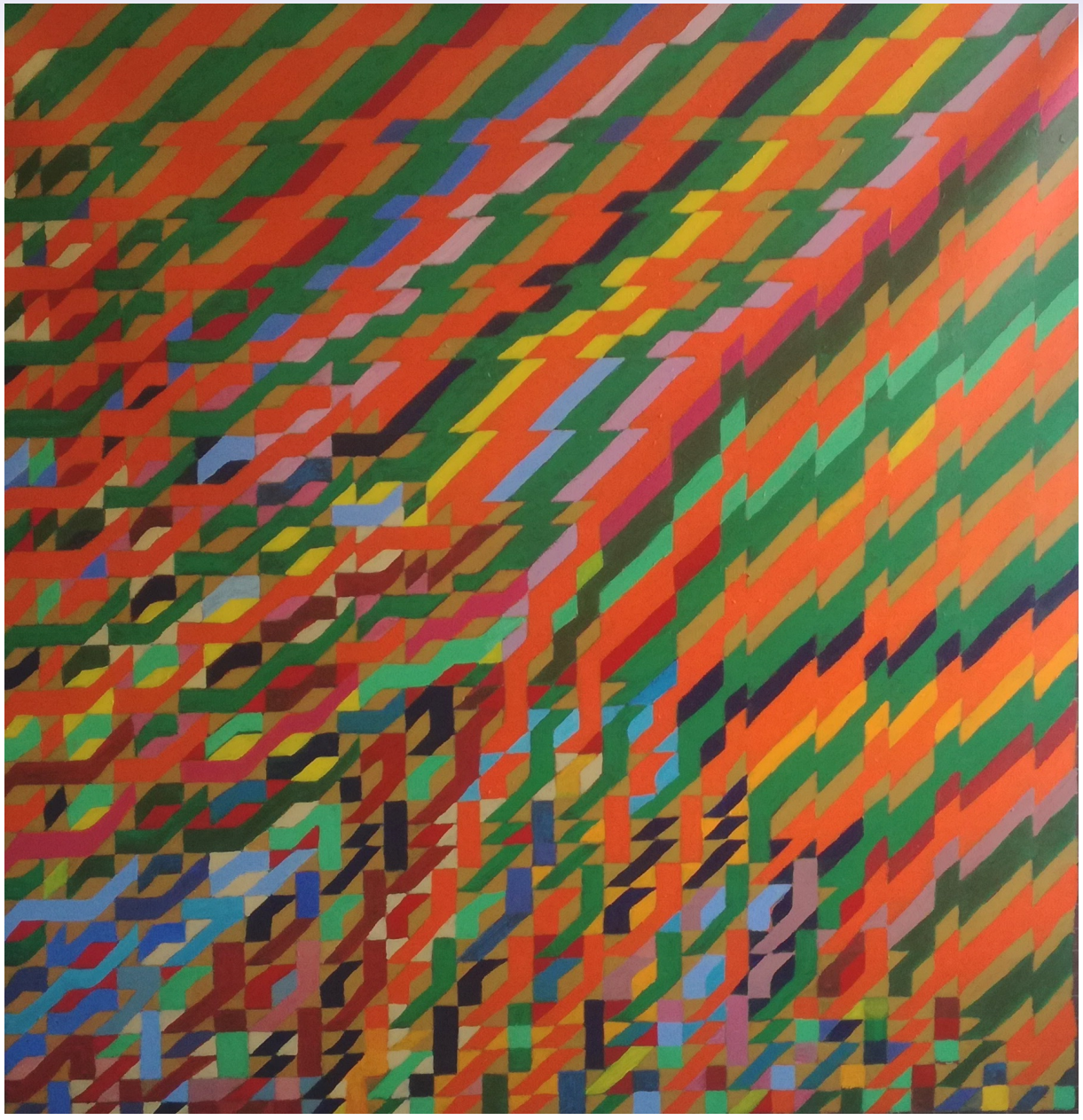
Coloring?



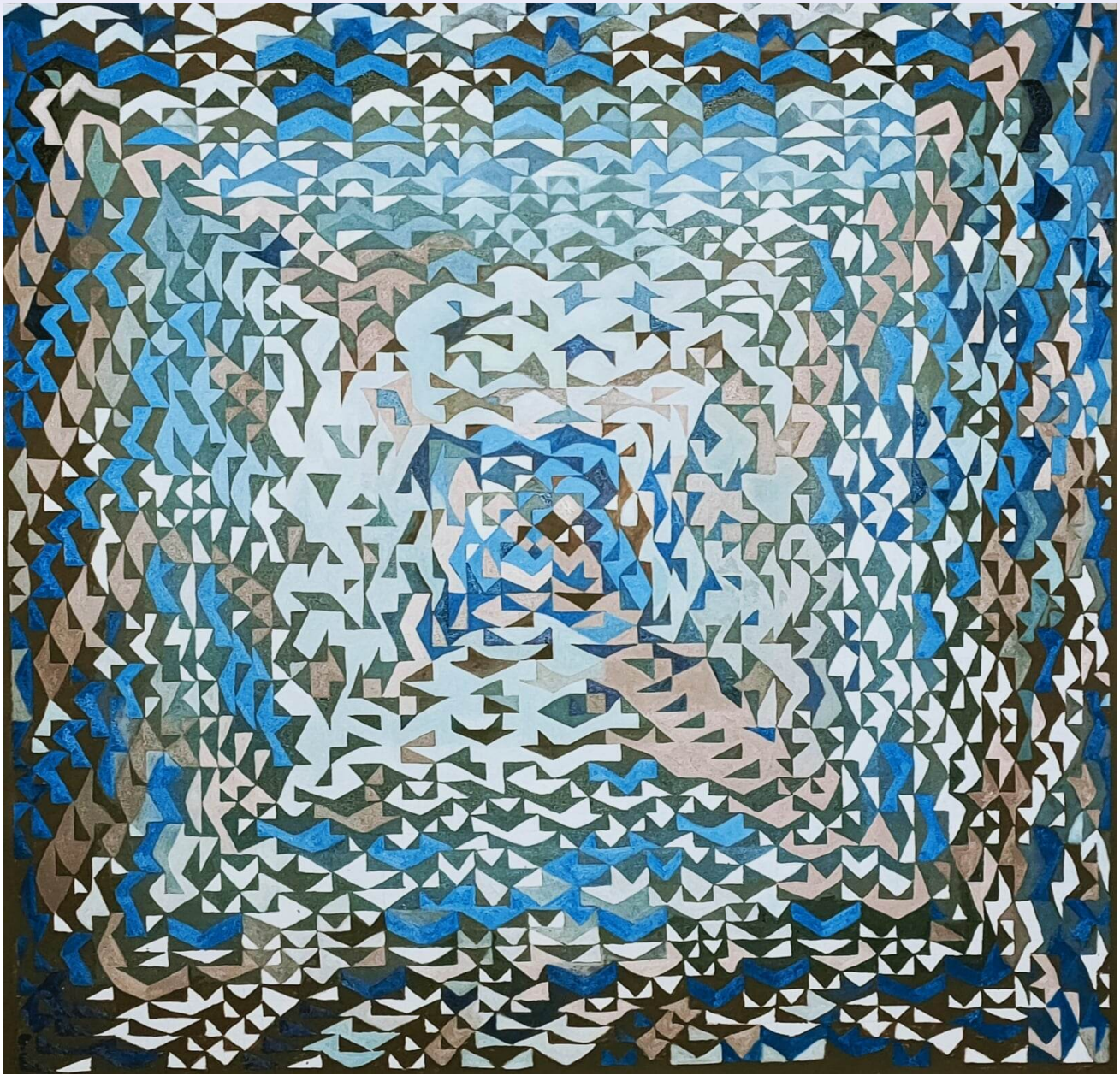
Golden Orbweaver – Reg Alcorn (2018)



Garden Raga – Reg Alcorn (2019)



Stripe Break – Reg Alcorn (2020)



Variations – Reg Alcorn (2023)

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Thank you!